XIX. *On the difference of Meridians of the Royal Observatories of Greenwich and Paris. By* THOMAS HENDERSON, *Esq. Communicated by* J. F. W. HERSCHEL, *Esq. Sec. R. S.*

Read May 17, 1827.

IN the Philosophical Transactions for 1826, Part II. Mr. HERSCHEL has given a detailed account of observations, which were made in the month of July, 1825, for the purpose of ascertaining the difference of the meridians of the Royal Observatories of Greenwich and Paris, with a computation of these observations, from which the most probable value of the difference of longitude appears to be $9^m\ 21^s{\cdot}6$. But I have perceived that in the copy of the observations delivered to him from the Royal Observatory of Greenwich, an error of one second has been committed; as the true sidereal time of the observation made there on 21st July, ought to be $17^h\ 38^m\ 5\ 7{\cdot}12$ in place of $17^h\ 38^m\ 56^s{\cdot}10$, set down in the Table p. 104, which he informs me was computed at the Observatory, and officially communicated to him from the Astronomer Royal. This error seems to have had its origin in the little Table at the bottom of page 103; for, on subtracting the error of the clock, $47^s{\cdot}37$, from the time $18^h\ 8^m\ 30^s{\cdot}40$, the true sidereal time is $18^h\ 7^m\ 43^s{\cdot}03$, instead of $18^h\ 7^m\ 42^s{\cdot}03$, there given. The error in the result of that day's observations, arising from this cause, has been partly compensated by a mistake of three tenths of a second, which

has occurred in calculating the combined observations of the same day, the gain of mean on sidereal time being stated to be $-4^s{\cdot}54$ (pp. 120 and 122), in place of $-4^s{\cdot}24$. On checking the other observations, a few trifling alterations appear to be necessary upon the Greenwich Table of sidereal time, from the *data* given along with it. These seem to be occasioned by different methods of calculation, and indeed are hardly worthy of notice. The French astronomers not having given the *data* on which the calculations of the sidereal times at Paris are founded, they are assumed to be correct.

The effect of the alterations thus made upon the elements of computation, is to redeem the result of the observations of 21st July from the suspicion which attached to it, in consequence of its disagreement with the results of the other days, and to produce a change of one-tenth of a second (corresponding to one hundred feet nearly on the earth's surface,) upon the most probable value of the difference of meridians forming the subject of investigation, it now appearing to be $9^m\ 21^s{\cdot}5$. At the same time the chronometer, which was employed for the observations at Fairlight down, is shown to have kept a more uniform rate than what previously appeared. In this important national operation, the utmost accuracy is desirable; and it has therefore been thought proper to subject the whole observations to a new computation. This will make more apparent the near coincidence of the results of the different days observations, and the great precision to be expected from experiments of the nature of those in question, when, as has happened in the present instance, they are

made with the utmost care and attention on the part of the observers.

The details of the new computation are as follows.

As mentioned by Mr. HERSCHEL, rockets were exploded at Wrotham, which were observed at Greenwich and Fairlight down; at La Canche, on the French Coast, which were observed at Fairlight and Lignieres; and at Mont Javoult, which were observed at Lignieres and Paris.

Before the difference of meridians can be eliminated from the observations, the rates of the chronometers employed at Lignieres and Fairlight must be ascertained. Let P be the sidereal time at Paris, L the corresponding time indicated by the chronometer at Lignieres (these times being determined by simultaneous observations of signals) P' and L', the same times for a subsequent night, then $(P' - P) - (L' - L)$ is the retardation of the chronometer at Lignieres on sidereal time during the chronometer time $(L' - L)$; $\frac{24^h \times [(P' - P) - (L' - L)]}{(L' - L)} = r$, is the rate of the chronometer, or the equation to be added to 24 hours of chronometer time to obtain the corresponding interval of sidereal time; and $\frac{r\,(L' - L)}{24^h}$ is the equation to be added to any portion of chronometer time $(L' - L)$ to reduce it to sidereal time.

In like manner with regard to the chronometer at Fairlight, let G and F be the corresponding sidereal time at Greenwich and chronometer time at Fairlight, G' and F' the same times for a subsequent night, then $\frac{24^h \times [(G' - G) - (F' - F)]}{(F' - F)} = r'$ is the chronometer's rate, or the

equation to be added to 24 hours of chronometer time, to obtain the corresponding interval of sidereal time; and $\frac{r'\,(F'-F)}{24^h}$ is the equation to be added to any portion of chronometer time $(F'-F)$, to reduce it to sidereal time.

Computation of the rates of the Chronometers.

1. Lignieres Chronometer.

From July 18 to July 19.

	h. m. s.		h. m. s.
19th P′ =	18 19 41·83	L′ =	10 29 33·97
18th P =	18 32 21·88	L =	10 46 13·60

$P'-P = 23\ 47\ 19{\cdot}95 \quad L'-L = 23\ 43\ 20{\cdot}37$

$(P'-P)-(L'-L) = 3^m\ 59^s{\cdot}58$

$$\frac{24^h \times 3^m\ 59^s{\cdot}58}{23^h\ 43^m\ 20^s{\cdot}37} = r = 4^m\ 2^s{\cdot}38.$$

From July 19 to July 20.

	h. m. s.	h. m. s.
20th	17 43 31·30	9 49 29·60
	18 3 32·55	10 9 27·80
	18 23 35·60	10 29 27·20
	18 43 42·15	10 49 30·85
	Mean	Mean
20th P′	= 18 13 35·40	L′ = 10 19 28·86
19th P	= 18 19 41·83	L = 10 29 33·97

$P'-P = 23\ 53\ 53{\cdot}57 \quad L'-L = 23\ 49\ 54{\cdot}89$

$(P'-P)-(L'-L) = 3^m\ 58^s{\cdot}68$

$$\frac{24^h \times 3^m\ 58^s{\cdot}68}{23^h\ 49^m\ 54^s{\cdot}89} = r = 4^m\ 0^s{\cdot}36.$$

From July 20 to July 21.

	h. m. s.		h. m. s.
21st P′ =	18 14 15·18	L′ =	10 16 14·18
20th P =	18 13 35·40	L =	10 19 28·86

$P'-P = 24\ 0\ 39{\cdot}78 \quad L'-L = 23\ 56\ 42{\cdot}62$

$(P'-P)-(L'-L) = 3^m\ 57^s{\cdot}16$

$$\frac{24^h \times 3^m\ 57^s{\cdot}16}{23^h\ 56^m\ 42^s{\cdot}62} = r = 3^m\ 57^s{\cdot}70.$$

From July 21 to July 22.

	h. m. s.		h. m. s.
22d P′ =	18 11 24·77	L′ =	10 9 24·63
21st P =	18 14 15·18	L =	10 16 11·48

$P'-P = 23\ 57\ 9{\cdot}59 \quad L'-L = 23\ 53\ 13{\cdot}15$

$(P'-P)-(L'-L) = 3^m\ 56^s{\cdot}44$

$$\frac{24^h \times 3^m\ 56^s{\cdot}44}{23^h\ 53^m\ 13^s{\cdot}15} = r = 3^m\ 57^s{\cdot}56.$$

2. Fairlight Chronometer.

From July 17 to July 18.

	h. m. s.	h. m. s.
17th	17 12 38·06	9 31 36·15
	17 22 39·55	9 41 35·90
	17 32 42·39	9 51 37·10
	17 42 44·31	10 1 37·50
	17 52 43·98	10 11 35·65
	18 2 49·09	10 21 38·85
	18 12 52·67	10 31 40·85
	18 22 54·52	10 41 40·95
	18 42 55·39	11 1 38·55
	Mean	Mean
17th	G = 17 53 53·33	F = 10 12 44·61
18th	G′ = 17 53 32·40	F′ = 10 8 28·13

G′ − G = 23 59 39·07 F′ − F = 23 55 43·52

$(G' - G) - (F' - F) = 3^m\ 55^s\cdot55$

$$\frac{24^h \times 3^m\ 55^s\cdot55}{23^h\ 55^m\ 43^s\cdot52} = r' = 3^m\ 56^s\cdot25.$$

From July 18 to July 19.

	h. m. s.	h. m. s.
19th	G′ = 18 12 20·31	F′ = 10 23 17·57
18th	G = 17 53 32·40	F = 10 8 28·13

G′ − G = 24 18 47·91 F′ − F = 24 14 49·44

$(G' - G) - (F' - F) = 3^m\ 58^s\cdot47$

$$\frac{24^h \times 3^m\ 58^s\cdot47}{24^h\ 14^m\ 49^s\cdot44} = r' = 3^m\ 56^s\cdot04.$$

From July 19 to July 20.

	h. m. s.	h. m. s.
20th	18 4 55·93	10 11 57·85
	18 15 4·26	10 22 4·45
	Mean.	Mean.
20th	G′ = 18 10 0·10	F′ = 10 17 1·15
19th	G = 18 12 20·31	F′ = 10 23 17·57

G′ − G = 23 57 39·79 F′ − F = 23 53 43·58

$(G' - G) - (F' - F) = 3^m\ 56^s\cdot21$

$$\frac{24^h \times 3^m\ 56^s\cdot21}{23^h\ 53^m\ 43^s\cdot58} = r' = 3^m\ 57\cdot25.$$

From July 20 to July 21.

	h. m. s.	h. m. s.
21st	G′ = 17 38 57·12	F′ = 9 42 7·65
20th	G = 18 10 0·10	F = 10 17 1·15

G′ − G = 23 28 57·02 F′ − F = 23 25 6·50

$(G' - G) - (F' - F) = 3^m\ 50^s\cdot52$

$$\frac{24^h \times 3^m\ 50^s\cdot52}{23^h\ 25^m\ 6^s\cdot50} = r' = 3^m\ 56^s\cdot25.$$

From July 21 to July 22.

	h. m. s.	h. m. s.
22d	G′ = 17 47 55·64	F′ = 9 47 8·59
21st	G = 17 38 57·12	F = 9 42 7·65

G′ − G = 24 8 58·52 F′ − F = 24 5 0·94

$(G' - G) - (F' - F) = 3^m\ 57^s\cdot58$

$$\frac{24^h \times 3^m\ 57\cdot58}{24^h\ 5^m\ 0^s\cdot94} = r' = 3^m\ 56^s\cdot76.$$

Collecting together the rates of the chronometers, we have

Lignieres Chronometer.

				r
From July 18th	to July 19th	- -	4ᵐ	2ˢ·38
19th	—— 20th	- -	4	0·36
20th	—— 21st	- -	3	57·70
21st	—— 22d	- -	3	57·56

Fairlight Chronometer.

				r′
From July 17th	to July 18th	- -	3ᵐ	56ˢ·25
18th	—— 19th	- -	3	56·04
19th	—— 20th	- -	3	57·25
20th	—— 21st	- -	3	56·25
21st	—— 22d	- -	3	56·76

As the rate of the Lignieres chronometer is irregular, it seems advisable to deduce the rate for reducing the observations there by interpolation from the two nearest rates, supposing that each answers to the middle of its interval. In this manner are obtained the following rates for the observations at Lignieres.

July 18th. $4^{m}\ 3^{s}{\cdot}41$; but as this differs considerably from the rate which the chronometer had on leaving Paris, the rate $4^{m}\ 2^{s}{\cdot}38$, derived from the observations of the 18th and 19th, has been retained, as being probably nearer the truth.

19th. $4^{m}\ 1^{s}{\cdot}38$.

21st. $3^{m}\ 57^{s}{\cdot}63$.

22d. $3^{m}\ 57^{s}{\cdot}49$.

The rate of the Fairlight chronometer being sufficiently uniform, the mean of the whole, $3^{m}\ 56^{s}{\cdot}51$, has been adopted for all the observations there, which cannot produce any sensible error.

The difference of meridians is now obtained from the following formula.

Let P be the sidereal time at Paris, L the corresponding chronometer time at Lignieres, L′ and F, corresponding chronometer times at Lignieres and Fairlight, and F′, chronometer time at Fairlight, and G the corresponding sidereal time at Greenwich; the intervals of chronometer times $(L' - L)$ and $(F' - F)$ must be reduced to intervals of sidereal time $(l' - l)$ and $(f' - f)$ by the formulæ given above. Then $P + (l' - l) + (f' - f) - G =$ difference of meridians required as is evident.

Computation of the difference of meridians; all the observers taken jointly.

July 18.

P	L	L′	F	F′	G.
h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.
18 15 40·37	10 29 34·40	9 54 52·00	9 46 29·75	9 41 46·20	17 26 46·17
18 35 41·13	10 49 32·80	10 14 54·00	10 6 31·40	9 51 49·60	17 36 50·97
18 45 44·13	10 59 33·60			10 1 50·30	17 46 53·59
				10 11 48·60	17 56 53·29
				10 21 46·90	18 6 53·41
				10 41 47·20	18 26 56·95
Mean	Mean	Mean	Mean	Mean	Mean
+18 32 21·88	10 46 13·60	10 4 53·00	9 56 30·57	10 8 28·13	− 17 53 32·40
+ 11 59·53	$= f' - f$			$l' - l =$	− 41 27·57
	$L' - L =$	− 41 20·60			
+18 44 21·41	Reduction =	− 6·97	$F' - F =$	+ 11 57·56	− 18 34 59·97
−18 34 59·97			Reduction =	+ 1·97	
	$l' - l =$	− 41 27·57			
9 21·44	= Diff. Merid^s		$f' - f =$	+ 11 59·53	

July 19.

P	L	L′	F	F′	G
h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.
17 29 29·60	9 39 30·40	9 44 49·40	9 36 33·10	9 42 0·45	17 30 56·55
18 39 52·50	10 49 41·10	9 54 49·90	9 46 33·65	9 51 53·65	17 40 51·34
18 49 43·40	10 59 30·40	10 34 49·70	10 26 33·70	10 1 56·50	17 50 55·76
		10 54 53·40	10 46 37·55	10 22 2·45	18 11 5·07
				10 32 24·75	18 21 28·63
				10 41 59·80	18 31 5·57
				10 51 59·60	18 41 7·08
				11 2 3·40	18 51 12·47
Mean	Mean	Mean	Mean	Mean	Mean
+18 19 41·83	10 29 33·97	10 17 20·60	10 9 4·50	10 23 17·57	−18 12 20·31
+ 14 15·41	$= f' - f$			$l' - l =$	− 12 15·42
	$L' - L =$	− 12 13·37	$F' - F =$	+ 14 13·07	
+18 33 57·24	Reduction =	− 2·05	Reduction =	+ 2·34	−18 24 35·73
−18 24 35·73					
	$l' - l =$	− 12 15·42	$f' - f =$	+ 14 15·41	
9 21·51	= Diff. Mer^s				

July 21.

P	L	L′	F	F′	G
h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.
17 37 23·10	9 39 24·70	9 54 50·40	9 46 38·95	9 42 7·65	17 38 57·12
17 47 32·10	9 49 32·70	10 4 53·10	9 56 41·50		
18 7 40·95	10 9 38·60	10 14 51·20	10 6 39·80		
18 17 30·30	10 19 26·40	10 34 49·60	10 26 38·25		
18 37 40·75	10 39 33·20	10 44 59·40	10 36 47·90		
18 57 43·90	10 59 33·30	11 4 51·80	10 56 41·10		
Mean	Mean	Mean	Mean	Mean	Mean
+18 14 15·18	10 16 11·48	10 26 32·58	10 18 21·25	9 42 7·65	—17 38 57·12
+ 10 22·81	= *l′ — l*			*f′ — f* =	— 36 19·55
	L′ — L =	+ 10 21·10	F′ — F =	— 36 13·60	
+18 24 37·99	Reduction =	+ 1·71	Reduction =	— 5·95	—18 15 16·67
—18 15 16·67					
	l′ — l =	+ 10 22·81	*f′ — f* =	— 36 19·55	
9 21·32	= Diff. Merids.				

July 22.

P	L	L′	F	F′	G
h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.	h. m. s.
17 31 12·15	9 29 18·60	9 44 50·80	9 36 39·80	9 32 8·95	17 32 53·31
17 51 18·70	9 49 21·90	9 54 53·50	9 46 42·45	9 42 6·95	17 42 53·38
18 1 15·65	9 59 17·10	10 4 53·10	9 56 42·50	9 52 8·55	17 52 56·58
18 11 21·70	10 9 21·70	10 15 8·60	10 6 57·85	10 2 9·90	18 2 59·29
18 21 43·60	10 19 41·70	10 24 48·30	10 16 37·50		
18 31 31·80	10 29 28·40	10 34 58·70	10 26 48·05		
18 51 29·80	10 49 23·00	10 44 57·50	10 36 47·00		
		10 54 47·70	10 46 37·15		
		11 4 48·80	10 56 37·90		
Mean	Mean	Mean	Mean	Mean	Mean
+18 11 24·77	10 9 24·63	10 24 54·11	10 16 43·36	9 47 8·59	- 17 47 55·64
+ 15 32·03	= *l′ — l*			*f′ — f* =	— 29 39·63
	L′ — L =	+ 15 29·48			
+18 26 56·80	Reduction =	+ 2·55	F′ — F =	— 29 34·77	- 18 17 35·27
—18 17 35·27			Reduction	= — 4·86	
	l′ — l =	+ 15 32·03			
9 21·53	= Diff. Merids.		*f′ — f* =	= —29 39·63	

These various results differ so little from each other, that their arithmetical mean, $9^m\ 21^s{\cdot}45$, may be assumed to be near the truth. But it may not be improper to ascertain the most probable value, and its probable error, by the calculus of probabilities as practised by GAUSS, BESSEL, &c. to serve as a rule for other investigations of a similar nature, in which it may be more requisite. Each night's result is liable to an error occasioned by the errors in the observed times of the signals, and of the transits of stars, whereby the clocks were compared with the heavens. The probable error of a single observation of a signal and a transit, appears from a considerable number of observations, to be one tenth of a second; and this divided by the square root of the number of these phenomena observed at any station, gives the probable error of the mean of the observed times at that station. But the results are exposed to other causes of error, such as the small deviations of the transit instruments from their meridians, the peculiar state of the eyes of the different observers, atmospheric circumstances, and various others which fluctuate from night to night, but may be supposed constant during the same night. Each night's result is equally liable to these errors, which have no tendency to be diminished by an increased number of observations upon that night. The probable error of each result arising from this cause is assumed to be two tenths of a second, and it is not likely to be more. Errors in the comparison of the chronometers employed for the observations of signals at Greenwich and Paris with the transit clocks, are also to be apprehended, which errors at each observatory may be supposed to be one tenth of a second. A probable error of one tenth of a second

in the reduction of the interval at Lignieres, on the 18th, ought also to be taken into account. By the theory of probabilities, the square of the probable error in the result of each night's observations is equal to the sum of the squares of the various errors, which by their combination produce the error in the result; and the weight to be attributed to each result is expressed by the reciprocal of the square of its probable error. The computation of these quantities is exhibited in the annexed Table, in which Δ denotes the difference of meridians obtained from each night's observations, e its probable error, and W its weight.

Day of Observation.	Δ	e^2	W.	$(\Delta - 9^m 21^s) \times$ W.
	m. s.	s.	s.	s.
18th	9 21·44	$\frac{0·01}{5}+\frac{0·01}{3}+\frac{0·01}{3}+\frac{0·01}{2}+\frac{0·01}{2}+\frac{0·01}{6}+\frac{0·01}{6}+\frac{0·01}{6}+0·04+0·01+0·01+0·01=$ 0·09366	10·68	4·699
19th	9 21·51	$\frac{0·01}{3}+\frac{0·01}{3}+\frac{0·01}{3}+\frac{0·01}{4}+\frac{0·01}{4}+\frac{0·01}{8}+\frac{0·01}{8}+\frac{0·01}{3}+0·04+0·01+0·01=$ 0·08083	12·37	6·309
21st	9 21·32	$\frac{0·01}{4}+\frac{0·01}{6}+\frac{0·01}{6}+\frac{0·01}{6}+\frac{0·01}{6}+\frac{0·01}{1}+\frac{0·01}{1}+\frac{0·01}{1}+0·04+0·01+0·01=$ 0·09917	10·08	3·226
22d	9 21·53	$\frac{0·01}{5}+\frac{0·01}{7}+\frac{0·01}{7}+\frac{0·01}{9}+\frac{0·01}{9}+\frac{0·01}{4}+\frac{0·01}{4}+\frac{0·01}{5}+0·04+0·01+0·01=$ 0·07408	13·60	7·208
			46·73)	21·442(0s·46

Most probable mean of the whole $= 9^m\ 21^s·46$.

The probable error of this mean is equal to the reciprocal of the square root of the sum of the weights; or $= \frac{1}{\sqrt{46·73}} = 0^s·15$.

It may therefore be said that $9^m\ 21^s·46$, or to the nearest tenth of a second, $9^m\ 21^s·5$ is the most probable value of the difference of meridians in question; that it is likely that this determination is within two tenths of a second of the truth; and that additional observations, even to a very considerable number, would not materially diminish the small uncertainty that still exists.

The above rectification of the observations made to ascertain the difference of longitude between Paris and Greenwich, not only adds greatly to the merit of the distinguished observers employed in the work, but trebles the value of their results by narrowing the extreme range of the experiments from $0^s{\cdot}65$ to $0^s{\cdot}21$.

THOMAS HENDERSON.

Edinburgh, 30th March, 1827.

ERRATA IN MR. HERSCHEL'S PAPER.

Page 85, July 20, *for* " 9 49^m $39^s{\cdot}6$ " *read* " 9^h 49^m $29^s{\cdot}6$."
19, — " 9 44 50" —— " 9 54 50."
90, 21, — " 10 26 28·3" —— " 10 26 38·3."
22, Signals No. 6, erroneously set down. As they stand they are repetitions of No. 7.
102, 21, *for* " clock 1^h 9^m $50^s{\cdot}88$" *read* " 17^h 9^m $50^s{\cdot}88$."
104, 17, — " —— 17 32 44·40" —— " 17 32 42·40."
— — — " —— 18 22 52·48" —— " 18 22 54·48."
109, line 10 from bottom, *for* " or" *read* " on."
111, — 7 from top, *for* " —" *read* " ×."
— — 12, ———— *for* " *z*" *read* " Z."
119, July 19, *for* " C′ 9^h 46^m $36^s{\cdot}65$" *read* " 9^h 46^m $33^s{\cdot}65$."
120, — 22, line 2d from bottom, *for* " — $3^s{\cdot}03$" *read* " $0^s{\cdot}03$.